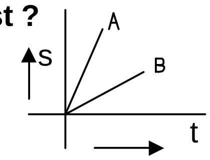
02. MOTION

Questions and Answers

1. "She moves at a constant speed in a constant direction." . Rephrase the same sentence in fewer words using concepts related to motion.

A. She moves with uniform velocity.

2. Distance Vs time graphs showing motion of two cars A and B are given. Which car moves fast ?



A. Speed = $\frac{Distance}{Time}$ = slope of the graph

Car 'A' travels more distance in less time.

It represents the slope of the curve of it.

So car 'A' moves fast.

3. Derive the equation for uniform accelerated motion for the displacement covered in its n^{th} second of its motion.

A. Distance travelled in 't' seconds

$$S = ut + \frac{1}{2}at^2$$

Distance travelled in 'n' seconds

$$S_n = un + \frac{1}{2}an^2$$

Distance travelled in '(n-1)' seconds

$$S_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2$$

$$S_{n-1} = un - u + \frac{1}{2}a(n^2\text{-}2n+1)$$

$$S_{n-1} = un - u + \frac{1}{2}an^2\text{-}an+\frac{1}{2}a$$

Distance travelled in **n^{th}** second

$$S_{n^{th}} = S_n - S_{n-1}$$

$$S_{n^{th}} = u + an - \frac{1}{2}a$$

$$S_{n^{th}} = u + a(n - \tfrac{1}{2}) \textbf{ or } u + \tfrac{1}{2}a(2n\text{-}1)$$

4. A body leaving a certain point " O" moves with an a constant acceleration. At the end of the 5^{th} second its velocity is 1.5 m/s. At the end of the sixth second the body stops and then begins to move backwards. Find the distance traversed by the body before it stops. Determine the velocity with which the body returns to point " O " ?

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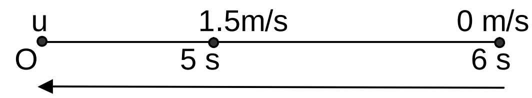
A.



$$V = u + at$$

Let initial velocity = 'u' m/s

Acceleration = '-a' m/s^2 (velocity decreases)

At the end of 5^{th} second V = 1.5 m/s

$$1.5 = u + (-a)5$$

$$1.5 = u - 5a \text{ ....(1)}$$

At the end of 6^{th} second V = 0 m/s

$$0 = u + (-a)6$$

$$0 = u - 6a$$

$$u = 6a \text{ ....(2)}$$

substitute (2) in (1): $1.5 = 6a - 5a$

$$a = 1.5 \text{ m/s}^2$$

from (2): $u = 6a = 6(1.5) = 9$ m/s

Distance travelled by body before it stops

(Distance travelled in t = 6 seconds)

$$S = ut + \frac{1}{2}at^2$$

$$S = (9)6 + \frac{1}{2}(-1.5)6^2$$

$$= 54 - (-1.5)18$$

$$= 54 - 27$$

$$S = 27 \text{ m}$$

When the body returns to point 'O'

u = 0 m/s ; a = 1.5 m/s^2 ; t = 6 s

Final velocity V = u + at

$$= 0 + 1.5(6)$$

$$= 9 \text{ m/s (in opposite direction)}$$

So V = -9 m/s

5. Distinguish between speed and velocity.

	Speed		Velocity
1	The distance travelled by the body in unit time.	1	The displacement of the body in unit time.
2	It is a scalar.	2	It is a vector.
3	Its value is always positive or zero.	3	Its value may positive or zero or negative.
4	Speed = $\frac{Distance}{Time}$	4	Velocity = $\frac{Displacement}{Time}$

6. What do you mean by constant acceleration?

A. If a body travels in straight line and its velocity changes (increase or decrease) by equal amount in equal time intervals, Then the acceleration is said to be uniform acceleration or constant acceleration.

7. A point mass starts moving in a straight line with constant acceleration "*a*". At a time t after the beginning of motion, the acceleration changes sign, without change in magnitude. Determine the time t from the beginning of the motion in which the point mass returns to the initial position.

A.

$$A \xrightarrow[t]{a} B$$

Let a particle starts at point "A"

The initial velocity $u = 0$ m/s

Acceleration $= a$ m/s^2

Displacement $= S$

Time taken to travel from A to B = t sec

According to first equation of motion

Final velocity $V = u + at$

$V = 0 + at$

$V = at$ …….(i)

According to second equation of motion

Displacement $S = ut + \frac{1}{2}at^2$

$S = 0(t) + \frac{1}{2}at^2$

$S = \frac{1}{2}at^2$ ……(ii)

After 't' time , the particle returned.

$$A \xleftarrow[t]{-a} B$$

Let a particle returns at point "B"

The initial velocity $u = at$ m/s

Acceleration $= -a$ m/s^2

Displacement $= -S$

Time taken to travel from B to A = t^l sec

According to second equation of motion

Displacement $S = ut + \frac{1}{2}at^2$

$-S = (at)(t^l) + \frac{1}{2}(-a)(t^l)^2$

$-S = att^l - \frac{1}{2}a(t^l)^2$

From (ii) we get

$-\frac{1}{2}at^2 = att^l - \frac{1}{2}a(t^l)^2$

$-\frac{1}{2}t^2 = tt^l - \frac{1}{2}(t^l)^2$

$-t^2 = 2tt^l - (t^l)^2$

$(t^l)^2 - 2tt^l - t^2 = 0$

This is in the form $ax^2 + bx + c = 0$

$x = \frac{-b \pm\sqrt{b^2-4ac}}{2a}$

$t^l = \frac{-(-2t) \pm\sqrt{(-2t)^2-4(1)(-t^2)}}{2(1)}$

$t^l = \frac{2t \pm\sqrt{4t^2+4t^2}}{2}$

$t^l = \frac{2t \pm\sqrt{8t^2}}{2}$

$t^l = \frac{2t \pm 2\sqrt{2}\,t}{2}$

$t^l = t + \sqrt{2}\,t$

Total time taken for travel $= t + t^l$

$= t + t + \sqrt{2}\,t$

$= 2t + \sqrt{2}\,t$

$= (2 + \sqrt{2})\,t$

8. Consider a train which can accelerate with an acceleration of 20cm/s^2 and slow down with deceleration of 100 cm/s^2. Find the minimum time for the train to travel between the stations 2.7 km apart.

A. If a body starting from rest, accelerates at the rate 'α' for some distance and decelerates at the rate 'β' and comes to rest after 't' seconds.

Average acceleration 'a' = $\left(\frac{\alpha\beta}{\alpha+\beta}\right)$

$\alpha = 20$ cm/s^2 ; $\beta = 100$ cm/s^2 ; t = ?

Distance S = 2.7 Km = 270000 cm

$ut + \frac{1}{2}at^2 = 270000$

$0(t) + \frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right)t^2 = 270000$

$\frac{1}{2}\left(\frac{20 \times 100}{20+100}\right)t^2 = 270000$

$\left(\frac{1000}{120}\right)t^2 = 270000$

$t^2 = 2700 \times 12$

$t^2 = 32400$

$t = 180$ sec

9. A train of length 50m is moving with a constant speed of 10m/s. Calculate the time taken by the train to cross an electric pole and a bridge of length 250 m.

A. Length of train = 50 m

Length of bridge = 250 m

Speed of train = 10 m/s

<u>In case of crossing an electric pole</u>:

Distance = length of train = 50 m

Time $= \frac{Distance\ travelled}{Speed\ of\ train} = \frac{50}{10} = 5$ sec.

In case of crossing a bridge:

Distance= length of train + length of bridge

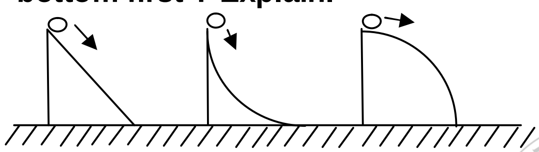
= 50 + 250 = 300 m

Time = $\frac{Distance\ travelled}{Speed\ of\ train} = \frac{300}{10}$= 30sec.

10. Correct your friend who says, " The car rounded the curve at a constant velocity of 70 km/h".

A. While a body is moving along a curved path, the direction changes continuously. So we should use speed instead of velocity. The correct statement is " The car rounded the curve at a constant speed of 70 $^{Km}/_{h}$.

11. Suppose that the three ball's shown in figure start simultaneously from the tops of the hills. Which one reaches the bottom first ? Explain.



A. The path which is to be travelled by the ball is short in first hill. So the ball from the top of the first hill reaches the ground.

12. When the velocity is constant, can the average velocity over any time interval differ from instantaneous velocity at any instant? If so, give an example; if not explain why?

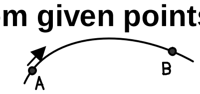
A. Constant velocity means both magnitude and direction are constant. If velocity is constant, the average velocity over any time interval is equal to instantaneous velocity at any time.

$$\overrightarrow{V_{average}} = \overrightarrow{V_{instant}}$$

13. Can the direction of velocity of an object reverse when it's acceleration is constant? If so give an example; if not, explain why?

A. In case of vertically projected body; at the maximum height of the body, the velocity is zero. It falls freely. It means its velocity was reversed. Acceleration due to gravity is constant and its value is 9.8 m/s^2.

14. As shown in figure, a point traverses the curved path.Draw the displacement vector from given points A to B.



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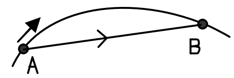
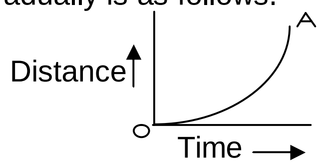
A. Displacement is the shortest distance in a specified direction. Displacement vector from A to B is denoted with $\overrightarrow{AB}$.



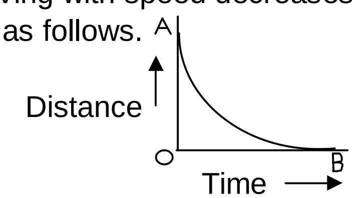
15. Draw the distance Vs time graph when the speed of a body increases uniformly

A. The distance – time graph for a body which is moving with speed increases gradually is as follows.



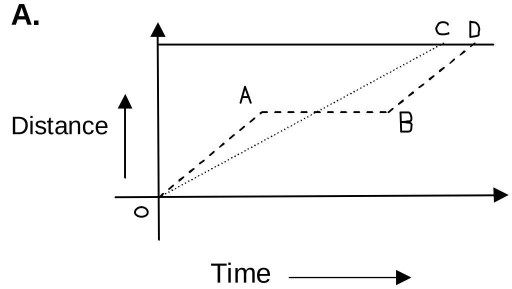
16. Draw the distance – time graph when its speed decreases uniformly.

A. The distance – time graph for a body which is moving with speed decreases gradually is as follows.



17. You may have heard the story of the race between the rabbit and tortoise. They started from same point simultaneously with constant speeds. During the journey, rabbit took rest some where along the way for a while. But the tortoise moved steadily with lesser speed and reached the finishing point before rabbit. Rabbit awoke and ran, but rabbit realized that the tortoise had won the race. Draw distance Vs time graph for this story.

A.



Hare and tortoise starts at "O". They reached final destination in different time. OC represents the motion of tortoise. OABD represents the motion of hare.

18. What is the average speed of a Cheetah that sprints 100m in 4sec. ? What if it sprints 50m in 2sec?

A. Case(i): distance = 100 m

Time = 4 s

Average speed = $\frac{Distance}{Time} = \frac{100}{4} = 25\ ^{m}/_{s}$

Case(ii): distance = 50 m

Time = 2 s

Average speed = $\frac{Distance}{Time} = \frac{50}{2} = 25\ ^{m}/_{s}$

19. A car travels at a velocity of 80 km/h during the first half of its running time and at 40 km/h during the other half. Find the average speed of the car.

A. Let us assume that the car travels for **2t** time.

for first 't' hours

Distance = velocity x time = 80 x t = 80t Km

for second 't' hours

Distance = velocity x time = 40 x t = 40t Km

Total distance travelled = 80t + 40t = 120t Km

Total time = '2t' hours

Average speed = $\frac{Total\ Distance}{Total\ Time}$

$= \frac{120t}{2t} = 60\ ^{Km}/_{h}$

20. A car covers half the distance at a speed of 50 km/h and the other half at 40km/h. Find the average speed of the car.

A. Let us assume that

The total distance is = '2s' Km

For travelling first 's' distance

speed = 50 $^{Km}/_{h}$

Time = $\frac{Distance}{Speed} = \frac{s}{50}$ hours

For travelling next 's' distance

speed = 40 $^{Km}/_{h}$

Time = $\frac{Distance}{Speed} = \frac{s}{40}$ hours

Total distance travelled = '2s' Km

Total time = $\left(\frac{s}{50} + \frac{s}{40}\right) = \frac{9s}{200}$ hrs

Average speed = $\frac{Total\ Distance}{Total\ Time}$

$= \frac{2s}{\left(\frac{9s}{200}\right)}$

$= \frac{400}{9} = 44.44\ ^{Km}/_{h}$

21. A particle covers 10m in first 5s and 10m in next 3s. Assuming constant acceleration. Find initial speed, acceleration and distance covered in next 2s.

A. Initial velocity = u $^{m}/_{s}$

Total distance S = 20 m

Total time t = 8 sec

Constant acceleration = a $^{m}/_{s^2}$

for 8 seconds $S = ut + \frac{1}{2}at^2$

$20 = u(8) + \frac{1}{2}a(8)^2$

$20 = 8u + 32a$

$2u + 8a = 5$ ………..(1)

for first 5 seconds $S = ut + \frac{1}{2}at^2$

$10 = u(5) + \frac{1}{2}a(5)^2$

$10 = 5u + \frac{25}{2}a$

$20 = 10u + 25a$

$2u + 5a = 4$ ………..(2)

Do (1) – (2): $3a = 1$

$a = \frac{1}{3}\ ^{m}/_{s^2}$ (acceleration)

Substitute this value in (1)

$2u + 8(\frac{1}{3}) = 5$

$2u = 5 - (\frac{8}{3})$

$2u = \frac{7}{3}$

$u = \frac{7}{6}\ ^{m}/_{s}$ (initial velocity)

Distance travelled in next 2 seconds =

$S_{10} - S_8 = \frac{7}{6}(10) + \frac{1}{2}(\frac{1}{3})(10)^2 - (20)$

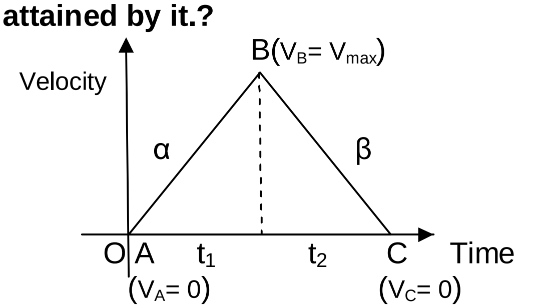
$= \frac{35}{3} + \frac{50}{3} - (20)$

$= \frac{85}{3} - 20$

$= \frac{25}{3} = 8.33$ m

22. A car starts from rest and travels with uniform acceleration "α" for some time and then with uniform retardation " β" and comes to rest. The time of motion is " t". Find the maximum velocity attained by it.?

A.



Velocity at 'A'(starting) = $V_A = 0$

Velocity at 'B'(maximum velocity) = $V_B = V_{max}$

Velocity at 'C'(ending) = $V_C = 0$

Acceleration = $\frac{Change\ in\ velocity}{Time}$

Let α is the acceleration from A to B and β is the deceleration from B to C

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$\alpha = \frac{V_B - V_A}{t_1} = \frac{V_{max} - 0}{t_1} = \frac{V_{max}}{t_1}$ $\rightarrow t_1 = \frac{V_{max}}{\alpha}$

$\beta = \frac{V_B - V_C}{t_2} = \frac{V_{max} - 0}{t_2} = \frac{V_{max}}{t_2}$ $\rightarrow t_2 = \frac{V_{max}}{\beta}$

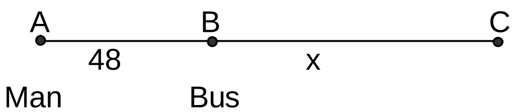
Let The total time taken by the car for acceleration and deceleration is 't'.

$t = t_1 + t_2 = \frac{V_{max}}{\alpha} + \frac{V_{max}}{\beta} = V_{max}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = V_{max}\left(\frac{\alpha+\beta}{\alpha\beta}\right)$

$$V_{max} = \left(\frac{\alpha\beta}{\alpha+\beta}\right)t$$

23. A man is 48m behind a bus which is at rest. The bus starts accelerating at the rate of 1 m/s^2, at the same time the man starts running with uniform velocity of 10 m/s. What is the minimum time in which the man catches the bus?

A.



Initial velocity of man u_m = 10 m/s

Initial velocity of bus u_b = 0 m/s

Acceleration of man a_m = 0 m/s^2 (uniform velocity)

Acceleration of bus a_b = 1 m/s^2

Let man starts at 'A' and bus starts at 'B'

And man catches the bus at 'C'.

Bus travels a distance 'x' before catching.

$$S = ut + \frac{1}{2}at^2$$

for the bus $x = (0)t + \frac{1}{2}(1)t^2$

$x = \frac{1}{2}t^2$ ....(1)

Man is 48m behind the bus.

He travels (48+x) meters before catching.

for the man $48 + x = (10)t + \frac{1}{2}(0)t^2$

$48 + x = 10t$ ....(2)

Substitute (1) in (2): $48 + \frac{1}{2}t^2 = 10t$

$96 + t^2 = 20t$

$t^2 - 20t + 96 = 0$

$t^2 - 12t - 8t + 96 = 0$

$t(t-12) - 8(t-12) = 0$

$(t-12)(t-8) = 0$

$t-12 = 0$ (or) $t-8 = 0$

$t = 12$ (or) $t = 8$

Minimum time for catching the bus is **8** s.

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24. Two trains, each having a speed of 30km/h, are headed at each other on the same track. A bird flies off one train to another with a constant speed of 60km/h when they are 60km apart till before they crash. Find the distance covered by the bird and how many trips the bird can make from one train to other before they crash?

A. Two trains are travelling opposite to each other on the same track.

Speed of each train = 30 Km/h

Relative speed of trains = 30 + 30 = 60 Km/h

The distance between two trains = 60 Km

The taken for train to reach together $= \frac{distance}{relative\ speed} = \frac{60}{60} = 1$hr

The speed of bird = 30 Km/h

Relative speed of bird with respect to trains =60+30=90 Km/h

The time to fly from first train to second train

(first half trip) $= t_1 = \frac{distance}{relative\ speed} = \frac{60}{90} = \frac{2}{3}$ hours

The distance travelled by two trains in $\frac{2}{3}$ hours

= relative speed x time = $60 \times \frac{2}{3} = 20 \times 2 = 40$ Km

The distance between two trains after $\frac{2}{3}$ hours =60–40=20 Km

The time to fly from second train to first train

(second half trip) $= t_2 = \frac{20}{90} = \frac{2}{9}$ hours $= \frac{2}{3^2}$ hours

The distance travelled by two trains in $\frac{2}{9}$ hours

= relative speed x time = $60 \times \frac{2}{9} = 20 \times \frac{2}{3} = \frac{40}{3}$ Km

The total time for one trip to bird

$= t_1 + t_2 = \frac{2}{3} + \frac{2}{9} = \frac{6}{9} + \frac{2}{9} = \frac{8}{9}$ hours $= \frac{8}{3^2}$ hours

The distance between two trains after $\frac{8}{9}$ hours (one trip)

$= 20 - \frac{40}{3} = \frac{20}{3}$ Km

The time to fly from first train to second train

(third half trip) $= t_3 = \frac{20/3}{90} = \frac{2}{27}$ hours $= \frac{2}{3^3}$ hours

The distance travelled by two trains in $\frac{2}{27}$ hours

= relative speed x time = $60 \times \frac{2}{27} = 20 \times \frac{2}{9} = \frac{40}{9}$ Km

The distance between two trains after $\frac{2}{27}$ hours

$= \frac{20}{3} - \frac{40}{9} = \frac{60}{9} - \frac{40}{9} = \frac{20}{9}$ Km

The time to fly from second train to first train

(fourth half trip) $= t_4 = \frac{20/9}{90} = \frac{2}{81}$ hours $\frac{2}{3^4}$ hours

The distance travelled by two trains in $\frac{2}{81}$ hours

= relative speed x time = $60 \times \frac{2}{81} = 20 \times \frac{2}{27} = \frac{40}{27}$ Km

The total time for second trip to bird

$= t_3 + t_4 = \frac{2}{27} + \frac{2}{81} = \frac{6}{81} + \frac{2}{81} = \frac{8}{81}$ hours $= \frac{8}{3^4}$ hours

The time taken to bird for travel successive half trips are

$\frac{2}{3}$ hours, $\frac{2}{3^2}$ hours, $\frac{2}{3^3}$ hours, $\frac{2}{3^4}$ hours ,...

The time taken to trains to reach together = 1 hour

The time for 'n' half trips to bird = 1 hour

$$\frac{2}{3} + \frac{2}{3^2} + \frac{2}{3^3} + \frac{2}{3^4} + \text{..........} + \frac{2}{3^n} = 1$$

$$\frac{2}{3}\left[1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \text{..........} + \frac{1}{3^{n-1}}\right] = 1$$

$$\frac{2}{3}\left[\frac{1-\left(\frac{1}{3}\right)^n}{1-\frac{1}{3}}\right] = 1$$

$$\frac{2}{3}\left[\frac{1-\left(\frac{1}{3}\right)^n}{\frac{2}{3}}\right] = 1$$

$$1 - \left(\frac{1}{3}\right)^n = 1$$

$$\left(\frac{1}{3}\right)^n = 0$$

n = infinite value.

So the bird can make infinite half trips. Means infinite trips.